6

Multi-stage events

Syllabus reference

Probability 3

- Multi-stage events

Statements about chance or probability are common in everyday life. A knowledge of probability can help in understanding sports, politics, science and other aspects of your life. This chapter focuses on drawing tree diagrams and using counting techniques.

ARE YOU READY?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillSHEET. Either click on the SkillSHEET icon next to the question on the *Maths Quest HSC Course* eBookPLUS or ask your teacher for a copy.

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Listing the sample space

Listing the sample space

1 List the sample space for each of the following events.
- **a** A card is drawn from a standard deck and its suit is noted.
- **b** A ball is selected from a bag containing three red, two blue and five white balls.
- **c** A pin is stuck in the page of a book and the nearest letter is noted.

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Informal description of chance

Informal description of chance

2 Describe each of the following events as being certain, probable, fifty–fifty, unlikely or impossible.
- **a** Winning the lottery.
- **b** Selecting an odd number from cards labelled with numbers 1 to 55.
- **c** Finding a $40 note in your wallet.

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Equally likely events

Equally likely events

3 For each of the events in question **2**, state whether or not each outcome is equally likely.

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Fundamental counting principle

Fundamental counting principle

4 In each of the following find the number of different ways each selection can be made.
- **a** One person is to be chosen from each of two classes with 20 people in one class and 25 in the other.
- **b** From a menu an entree is to be chosen from a selection of five entrees followed by a main course from a selection of eight, and then a dessert from a selection of six.
- **c** Car number plates consisting of two letters, followed by two digits, followed by another two letters.

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Single event probability

Single event probability

5 Find the probability of each of the following events.
- **a** Randomly selecting the winner of a swimming final with eight competitors.
- **b** Winning a raffle when 150 tickets are sold and you purchase three tickets.
- **c** Selecting a $2 coin from a pocket containing three $2 coins, four $1 coins and seven 20c pieces.

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Determining complementary events

Determining complementary events

6 Find the complement to each of the following events.
- **a** Selecting a vowel from the letters of the alphabet.
- **b** Choosing a black marble from a bag with 12 black, 23 white and 15 clear marbles.
- **c** Selecting a number less than 10.

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Calculating the probability of a complementary event

Calculating the probability of a complementary event

7 Find the probability of:
- **a** winning a football match given the probability of losing is $\frac{2}{5}$.
- **b** the train being late given that it is on time four days out of every five.
- **c** a golfer missing a putt given the probability of sinking the putt is 0.73.

6A Tree diagrams

If an event has more than one stage to it, then a tree diagram can be drawn to list the sample space accurately. In a tree diagram the tree branches out once for each stage of the experiment. At each stage the number of branches is the same as the number of possible outcomes.

To list the sample space we then follow the tree to the end of each branch and record the outcome at each stage.

WORKED EXAMPLE 1

A coin is tossed three times. Draw a tree diagram and use it to list the sample space for this experiment.

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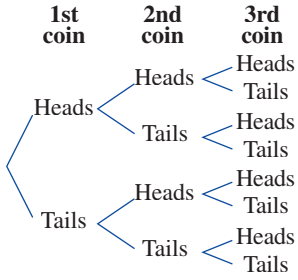
Drawing a tree diagram

THINK

1. There are three stages to the experiment.
2. At each stage the outcome can be heads or tails.
3. Draw the tree diagram branching out three times with two branches at each stage.

WRITE



4. List the sample space by following the path to each end branch.

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

In the above example, each stage of the experiment (each toss of the coin) is independent of the other stages. That is to say, the outcome of one toss does not affect the outcome of another toss. In many examples, the outcome of one stage will affect the outcome of another. Consider worked example 2. Here we are forming a two-digit number such that no digit may be repeated. Once a number has been chosen as the first digit, it can not be chosen as the second digit. Therefore, the first stage of the experiment does affect the second stage.

WORKED EXAMPLE 2

A two-digit number is formed using the digits 4, 5, 7 and 9 without repetition. Draw a tree diagram and use it to list all possible numbers that can be formed.

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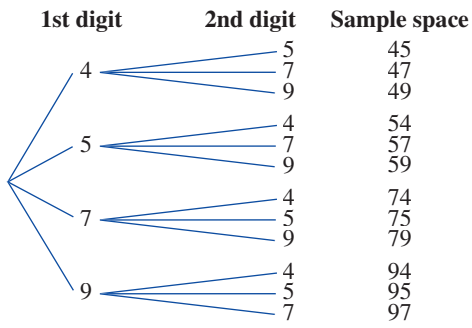
Tree diagram with restrictions

THINK

1. There are two stages to the experiment.
2. For the first stage there will be four branches and since one number is chosen there will be three branches for the second stage.
3. Draw the tree diagram.
4. List the sample space by following the branches to each end point on the tree diagram.

WRITE



Once the tree diagram is completed, the probability of an event can be calculated using the formula:

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

WORKED EXAMPLE 3

A coin is tossed and a die is rolled. Calculate the probability of tossing a tail and rolling a number greater than 4.

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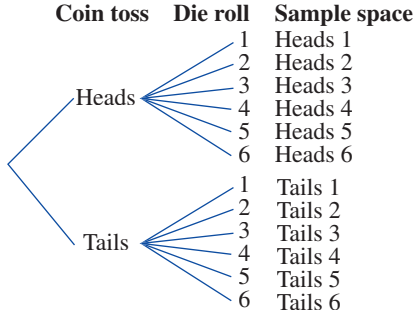
Tossing a coin and rolling a die

THINK

1. There are two stages to the event.
2. At the first stage there are two outcomes and at the second stage there are six outcomes.
3. Draw the tree diagram.

WRITE



4. List the sample space by following the branches to each end point on the tree diagram.
5. Calculate the probability using the probability formula. There are two favourable outcomes — T5 and T6.
6. Simplify.

$\text{P(tail and no.} > 4) = \frac{2}{12}$

$= \frac{1}{6}$

REMEMBER

1. In any probability experiment that has more than one stage, a tree diagram can be used to calculate the sample space.
2. The tree diagram branches once for each stage and the number of branches at each stage is equal to the number of outcomes.
3. The sample space is found by following the path to the end of each branch.
4. Once the sample space has been found, the probability of each outcome is calculated using the probability formula:

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

EXERCISE 6A

Tree diagrams

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Listing the sample space

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Informal description of chance

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Equally likely events

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Tree diagrams

1 WE1 A family consists of four children. Draw a tree diagram to show all possible combinations of boys and girls.

2 Two dice are cast. Draw a tree diagram that will allow you to list the sample space of all possible outcomes.

3 There are two bags each containing a red, blue, yellow and green marble. One marble is to be chosen from each bag. Draw a tree diagram that will allow you to calculate the sample space.

4 A school is to send one male and one female representative to a conference. The boys nominate George, Frank, Stanisa and Ian; the girls have nominated Thuy, Petria, Joan, Wendy and Amelia. Draw a tree diagram and list the sample space for all possible choices of representatives.

5 WE2 A two-digit number is to be formed using the digits 1, 2, 4, 5 and 7 such that no digit may be repeated. Draw a tree diagram to list all possible numbers that can be formed.

6 A committee needs to elect a president, secretary and treasurer. The four nominations for these positions are Belinda, Dean, Kate and Adrian. Given that no person is allowed to hold more than one position, use a tree diagram to list all ways in which these three positions can be filled.

7 The digits 3, 5, 7 and 8 are used to form a three-digit number. If no digit can be used more than once list the sample space.

8 MC From a group of five nominations a school captain and vice-captain are to be elected. The number of ways that the captain and vice-captain can be chosen is:

A 5 **B** 10 **C** 20 **D** 25

9 WE3 The four aces from a deck of cards are placed face down on a table. One card is chosen followed by a second card without the first card being replaced. Calculate the probability that the ace of hearts is one of the two cards chosen.

10 A two-digit number is formed using the digits 2, 3, 4 and 7 without repetition.
- **a** Use a tree diagram to list the sample space.
- **b** Calculate the probability that the number formed is greater than 35.

11 A tennis team consists of three men, Andre, Yevgeny and Jonas and two women, Martina and Lindsay. From the team the captain and the vice-captain are to be chosen. Calculate the probability that the captain and vice-captain are:
- **a** Andre and Lindsay
- **b** both men
- **c** the same sex
- **d** different sex.

12 Find the probability that all three children in a family will be the same sex.

13 MC A three-digit number is formed using the digits 5, 6, 8 and 0. No digit can be repeated and the 0 can't be first. The probability of the number formed being greater than 800 is:

A $\frac{1}{4}$ **B** $\frac{1}{3}$ **C** $\frac{3}{16}$ **D** $\frac{1}{2}$

14 An airline offers holidays to three destinations: Brisbane, Gold Coast or Cairns. The holiday can be taken during two seasons: Peak season or Off-peak season. The customer has the choice of three classes: Economy, Business or First class. There is no First class to Cairns, however.

a Use a tree diagram to list all combinations of holiday that could be taken by choosing a destination, season and class.

b Terry takes a mystery flight, which means he is allocated a ticket at random from the above combinations. Calculate the probability that Terry's ticket:

i goes to Brisbane

ii is First class

iii is in Peak season, flying First class.

Further development

15 A three-digit number is to be formed using the digits 5, 7, and 8.

a If the same digit cannot be used twice, how many three-digit numbers can be formed?

b If repetition is allowed, how many numbers can be formed?

c Kevin claims that the probability of an even number will be the same regardless of whether repetition of digits is allowed or not. Is Kevin correct? Explain why or why not.

16 In a family of four children there can be

- more boys
- more girls
- an equal number of boys and girls.

a Is each of these outcomes equally likely to occur?

b Dan claims that if there is an even number of children the probability of there being an equal number of boys and girls is the same. Is Dan correct? Explain your answer.

17 Give a brief explanation of why, when two dice are rolled, that each total is not equally likely to occur.

18 Tanya is allergic to peanuts. On a menu there are 4 entrees, 6 main courses and 4 desserts. Of these, 2 entrees, 3 main courses and 2 desserts contain traces of peanuts. Tanya says that that the number of combinations of meals that she may choose is halved. Is Tanya correct? Explain your answer.

19 Ingrid tosses a coin 10 times and claims that there is exactly a 90% chance that she will throw at least one Head. Is Ingrid correct? Explain your answer.

6B Counting techniques

Ordered arrangements

1 Select three people and stand them in a line.
2 Now get the three people to stand in a different order.
3 In how many different orders can the three people be placed?
4 Repeat the above process with four people in the line.
5 Is there a pattern? Can you calculate the number of different ways in which five people can be arranged?

There are 10 people standing in a line. In how many ways can they be arranged? To calculate this we need to consider the number of ways that each place in the line can be filled. To do this we need to calculate the number of people remaining after we fill each place in the line.

- There are 10 people who could fill the first position.
- Once the first position has been filled, there are nine people remaining to fill the second position.
- Once the second position has been filled, there are eight people remaining to fill the third position.
- This pattern continues until there is only one person left who can fill the last position.

Calculating this: $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 3\,628\,800$.

A shorter way of writing $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ is to write 10!, that is, 10 factorial.

Your calculator will have a factorial function, usually labelled $x!$. Make sure that you know where this function is on your calculator.

WORKED EXAMPLE 4

Calculate the value of 8!.

THINK	WRITE
Enter 8 and press 2ndF $x!$ on the calculator.	$8! = 40\,320$

WORKED EXAMPLE 5

Six people are standing in a line. In how many ways can the six people be arranged?

THINK	WRITE
1 The answer is 6!	$6! = 720$
2 Give a written answer.	The people can be arranged in 720 ways.

Tree diagrams and ordered arrangements

Four people, Anji, Belinda, Kristen and Summer, are to be placed in order.

1 Calculate the number of different ways these four girls can be placed in a line.
2 Draw a tree diagram and use it to list the ways that the four girls can be placed in order.
3 Check that the number of elements in the sample space found from your tree diagram corresponds to the answer obtained in part 1.

In worked examples 4 and 5, we have been ordering an entire group. In some cases we may wish to order only part of the group. Consider the case of an Olympic swimming final. There are eight swimmers and we wish to know the number of ways that the gold, silver and bronze medals can be awarded.

- There are eight possible winners of the gold medal.
- With the first place filled, there are seven possible winners of the silver medal.
- With both first and second places filled, there are six possible winners of the bronze medal.

$$\text{Calculating this: number of arrangements} = 8 \times 7 \times 6$$
$$= 336$$

This type of arrangement is known as an ordered selection. It occurs when the order in which the choices are made is important. In the worked example below, a captain and a vice-captain are to be chosen. If Benito is captain and Imran is vice-captain, this is a different selection to Imran as captain and Benito as vice-captain.

To calculate the number of ordered selections that can be made, we multiply, starting from the number of possible first selections, then reducing by one with each multiplication until each position is filled.

WORKED EXAMPLE 6

In a cricket team of eleven players, a captain and vice-captain are to be chosen. In how many ways can this be done?

THINK	WRITE
1 There are 11 possible choices of captain.	
2 Once the captain is chosen, there are 10 choices remaining for vice-captain.	Number of arrangements $= 11 \times 10$ $= 110$

Committee selections

On a committee of five people, a president and a vice-president are to be chosen. The five committee members are Andreas, Brett, Cathy, Dharma and Emiko.

1 Use the method shown in worked example 6 to calculate the number of ways in which the president and the vice-president can be chosen.
2 Now use a tree diagram to list the sample space of all possible selections of president and vice-president.
3 Check that the number of elements in the sample space corresponds to the answer obtained in part 1 of this investigation.

Consider a case where two representatives to a committee are chosen from a class of 20 students. This is an example of an unordered selection. If Sue is chosen, followed by Graham, this is the same choice as if Graham is chosen and then Sue.

To calculate the number of unordered selections that can be made, we calculate the number of ordered selections that can be made and then divide by the number of arrangements of these selections. This is calculated using factorial notation as in worked example 5. In the case of choosing the committee:

Number of ordered selections is $20 \times 19 = 380$.
Two people can be arranged in two (2!) ways.
Number of unordered selections $= 380 \div 2$
$= 190$

WORKED EXAMPLE 7

From a group of eight athletes, three are to be chosen to represent the club at a carnival. In how many ways can the three representatives be chosen?

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Worked example 7

THINK	WRITE
1 Calculate the number of ordered selections that can be made.	Ordered selections = $8 \times 7 \times 6$ = 336
2 Calculate the number of arrangements of the representatives.	Arrangements = $3 \times 2 \times 1$ = 6
3 Divide the ordered selections by the arrangements of the representatives.	Unordered selections = $336 \div 6$ = 56

Unordered selection

A rowing team has six members: Mark, Norman, Olaf, Pieter, Quentin and Raymond. Two are to be chosen to be the crew in a pairs race.

1 Use the method described in worked example 7 to calculate the number of pairs that could be chosen.
2 Use a tree diagram to list the ordered selections and then write the sample space of unordered selections by ignoring any repeated pair.
3 Check that the number of elements of the sample space corresponds to the answer obtained in part 1 of this investigation.

REMEMBER

1. A group of n different items can be arranged in $n!$ ways.
2. $n! = n \times (n-1) \times (n-2) \times \ldots \times 1$ and can be found as a function on your calculator.
3. When an ordered selection is made, the number of selections can be calculated by multiplying the number of first choices that can be made by the number of second choices that can be made and so on.
4. To calculate the number of unordered selections that can be made, we divide the number of ordered selections by the number of arrangements of those selected.

EXERCISE 6B

Counting techniques

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Fundamental counting principle

1 WE4 Use your calculator to calculate the value of the following.
a 3! **b** 5! **c** 9!

2 WE5 Four people are involved in a race. In how many different orders can they complete the race?

3 The letters A, B, C, D and E are written on cards. In how many different orders can the cards be placed?

4 A three-digit number is formed using the digits 3, 6 and 8. If no number can be repeated, how many numbers is it possible to form?

5 In a race of 10 people, in how many different ways can the first three places be filled?

6 WE6 In a school, a captain and vice-captain are to be elected. The four nominations are Geri, Reika, Melanie and Victoria. In how many different ways can the captain and vice-captain be chosen?

7 In the Melbourne Cup there are 24 horses. In how many different ways can the three placings be filled?

8 WE7 Seven people try out for three places on a debating team. In how many ways can the team of three be chosen from the group of seven?

9 How many different groups of four can be selected from ten people?

10 In his pocket Trevor has six coins: a $2 coin, $1 coin, 50c coin, 20c coin, 10c coin and 5c coin. If Trevor randomly chooses two coins, how many different sums of money are possible?

11 On a restaurant menu there is a choice of three entrees, six main courses and four desserts. In how many ways can a person choose an entree, main course and dessert from the menu?

12 MC Which of the following is an example of an unordered selection?

- **A** Five students are placed in order of their exam results.
- **B** From a group of five students, a contestant and a reserve are chosen for a Mathematics competition.
- **C** From a group of five students, two are chosen to represent the class on the SRC.
- **D** From a group of five students, two are awarded 1st and 2nd prizes in Mathematics.

13 MC The numbers 1, 2, 3 and 4 are used to form a three-digit number such that no digit can be used more than once. The number of three-digit numbers that can be formed is:

A 4 **B** 6 **C** 12 **D** 24

14 MC Gavin, Dion, Michael, Owen and Shane try out for two places on a tennis doubles team. The number of teams that can be chosen is:

A 5 **B** 10 **C** 20 **D** 25

15 A small play has three characters. Six people, Wendy, Rebecca, Thai, Yasmin, Andrea and Ophelia, audition for the three parts.

- **a** How many different groups of three can be chosen for the play?
- **b** In how many different ways can the three parts be allocated to the three girls?

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16 At the Olympic qualifying trials, nine cyclists compete for a place on the team.

- **a** In how many different orders can the competition finish?
- **b** How many different ways can 1st, 2nd and 3rd place be filled?
- **c** Two cyclists are chosen to represent Australia on the team. How many different teams of two can be chosen?

Further development

17 A rowing crew consists of 4 rowers who sit in a definite order. How many different crews are possible if 5 people try out for selection?

18 The school musical needs a producer, director, musical director and script coach. Nine people have volunteered for any of these positions. In how many different ways can the positions be filled? *Note:* One person cannot take on more than 1 position.

19 There are four people in a race. Explain why the number of ways that the first three places can be filled is the same as the number of ways that all four places can be filled.

20 A rugby union squad has 12 forwards and 10 backs in training. A team consists of 8 forwards and 7 backs. How many different teams can be chosen from the squad?

21 Lotto is a gambling game played by choosing 6 numbers from 45. Gamblers try to match their choice with those numbers chosen at the official draw. No number can be drawn more than once and the order in which the numbers are selected does not matter.

- **a** How many different selections of 6 numbers can be made from 45?
- **b** Suppose the first numbers drawn at the official draw are 42, 3 and 18. How many selections of 6 numbers will contain these 3 numbers?
- **c** Suppose the first numbers drawn at the official draw are 42, 3 18 and 41. How many selections of 6 numbers will contain these 4 numbers?
  Note: This question ignores supplementary numbers.

22 Explain why the number of ways that three people can be selected from 10 is the same as the number of ways that seven people can be selected from 10.

6C Probability and counting techniques

Once the counting techniques done in the previous section have been completed, we can calculate the probability of certain events occurring. To do this we go back to using the probability formula:

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

WORKED EXAMPLE 8

The letters A, H, M, S and T are written on cards. The cards are shuffled and then laid out face up. Calculate the probability that the cards form the word MATHS.

THINK	WRITE
1 The five cards can be arranged in 5! ways.	No. of arrangements $= 5!$ $= 5 \times 4 \times 3 \times 2 \times 1$ $= 120$
2 MATHS is one way of arranging the letters and so we use the probability formula.	$\text{P(MATHS)} = \frac{1}{120}$

We also need to be able to calculate the probability of a particular ordered or unordered arrangement occurring.

WORKED EXAMPLE 9

From Francis, Gary, Harley, Ike and Jacinta, a school captain and vice-captain need to be elected. Calculate the probability that Ike and Jacinta occupy the two positions.

THINK	WRITE
1 Calculate the number of ordered selections that are possible.	No. of ordered selections $= 5 \times 4$ $= 20$
2 Ike and Jacinta in the two positions can be arranged in two ways.	No. of arrangements $= 2 \times 1$
3 Divide the ordered selections by the number of arrangements.	No. of unordered selections $= 20 \div 2$ $= 10$
4 Substitute into the probability formula.	P(Ike and Jacinta) $= \frac{1}{10}$

WORKED EXAMPLE 10

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Worked example 10

A bag contains a red, green, yellow, blue, orange and purple marble. Three marbles are selected from the bag. Calculate the probability that the red, yellow and orange marbles are chosen.

THINK	WRITE
1 Calculate the number of ordered selections.	No. of ordered selections $= 6 \times 5 \times 4$ $= 120$
2 Calculate the number of arrangements.	No. of arrangements $= 3 \times 2 \times 1$ $= 6$
3 Calculate the number of unordered selections.	No. of unordered selections $= 120 \div 6$ $= 20$
4 The red, yellow and orange marble is one possible selection.	
5 Substitute into the probability formula.	P(red, yellow and orange) $= \frac{1}{20}$

Popular gaming

There are many different forms of lottery that depend upon ordered or unordered arrangements.

1. Lotto — This requires the player to select six numbers out of 45. In how many ways can the six numbers be chosen? Remember order does not matter.
2. Similar games to Lotto are:
   - a. Oz Lotto — seven numbers are chosen from 45.
   - b. The Pools — six numbers are chosen from 38.

   In how many ways can the six numbers for each of these games be chosen?
3. Powerball — This requires the player to choose five numbers from 45 in an unordered selection. A sixth ball (the powerball) is chosen from a second barrel containing 45 balls. In how many ways can this be selected?
4. Lotto Strike — The player must select the first four balls drawn from 45 in the correct order. In how many ways can this ordered selection be made?

REMEMBER

When we have calculated the number of arrangements and the number of ordered or unordered selections that are possible, we can then calculate the probability of a certain selection using the probability formula.

EXERCISE 6C

Probability and counting techniques

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Single event probability

1 WE8 Four people, Craig, Barry, Anne and Dimitri, are arranged in a line. Calculate the probability that the four people are arranged in alphabetical order.

2 The numbers 4, 5, 6, 7 and 8 are arranged to form a five-digit number such that no digit can be repeated. Calculate:
- **a** how many five-digit numbers can be formed
- **b** the probability that the number formed is 54 867
- **c** the probability that the number formed is 86 574.

3 A three-digit number is formed using the digits 6, 8 and 9 and no digit may be repeated. Calculate the probability that the number formed is:
- **a** 896
- **b** even
- **c** greater than 800.

4 WE9 There are five candidates in an election for SRC president. The second placed candidate will be made vice-president of the SRC. If Lauren and Meta are two of the candidates, calculate the probability that they will occupy the two positions.

5 Seven surfers enter a competition. If two of the surfers are Kurt and Paul, calculate the probability that:
- **a** Kurt comes first and Paul comes second
- **b** Paul comes first and Kurt comes second
- **c** Kurt and Paul fill the first two places.

6 From the digits 1 to 9 a two-digit number is formed such that no digit can be repeated. Calculate the probability that the number formed is:
a 67 **b** greater than 80 **c** less than 30.

7 WE10 From a deck of cards, the four aces are laid face down on a table. Two of the aces are then turned face up. Calculate the probability that the two aces turned face up are the ace of clubs and the ace of spades.

8 An ice-cream parlour offers a choice of 25 flavours. A triple scoop ice-cream places three different flavours on top of each other. If the flavours are chosen randomly, find the probability that the ice-cream is:
a vanilla, chocolate and strawberry in that order
b vanilla, chocolate and strawberry in any order.

9 Six boys try out for three places on a debating team. The boys are Gavin, David, Andrew, Rhyse, Julius and Elliot.
a How many teams of three is it possible to choose?
b Calculate the probability that Gavin, Andrew and Elliot are on the team.

10 The letters M, A, I, D and G are written on cards and two of these are to be chosen. Calculate the probability that the two cards chosen are:
a both vowels **b** both consonants **c** one vowel and one consonant.

Further development

11 To win Lotto you must correctly select the six correct winning numbers from a possible 45 numbers.
a Find the probability of winning Lotto when selecting a single set of six numbers.
b To increase their chances of winning some players take a systems entry. This means selecting more than six numbers. For example, a system 7 is where 7 numbers are chosen and is equivalent to all combinations of six numbers that can be chosen within 7. Find the probability of winning Lotto if:
i a system 7 entry is played
ii a system 8 entry is played
iii a system 15 entry is played.

12 A second game that is played within Lotto is called Lotto Strike. This involves selecting the first four Lotto balls drawn in the correct order. Determine whether Lotto or Lotto Strike is harder to win.

13 Oz Lotto is another similar game where the player has to select seven numbers from 45.
- John thinks that, because you need to select one extra ball, Oz Lotto will be $\frac{1}{7}$ or 14.3% harder to win.
- Peter thinks that, because you need to select one extra ball, Oz Lotto will be $\frac{1}{6}$ or 16.7% harder to win.
- Bruce thinks that both Peter and John are incorrect and that Oz Lotto will be about 450% harder to win.

Who is correct? Explain your answer.

14 Powerball is a game where the player must select five numbers out of 45 from one barrel and then a single ball out of 45 from a second barrel.
a Find the number of ways that the first set of five numbers can be selected.
b Find the number of ways that the single number can be selected.
c Find the probability of winning Powerball with a single entry.

15 Keno is a popular game where 20 numbers are selected from 80. The major jackpot is won by a player selecting 10 numbers and those 10 numbers being among the 20 drawn.

a In approximately how many ways can the player select 10 numbers from 80? Give your answer in scientific notation correct to 3 significant figures.

b How many winning combinations of 10 numbers from 20 are there?

c Find the probability of selecting a winning combination in scientific notation correct to 3 significant figures.

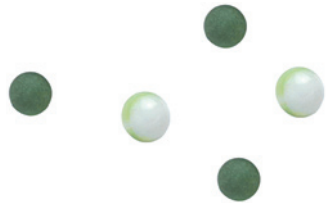
16 In the game of Keno find the approximate probability, as a decimal, of winning the:

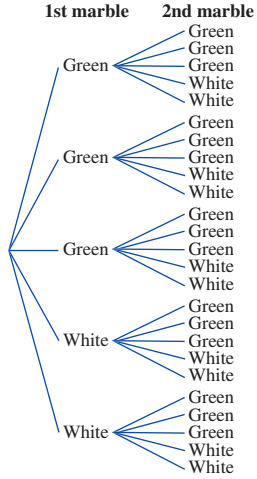
a 2 number game **b** 3 number game **c** 5 number game.

6D Probability trees

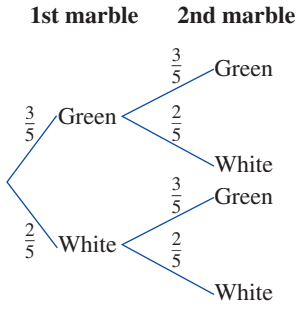
In the tree diagrams studied so far, the probability of each outcome has been equally likely. When each result is not equally likely we can still draw the diagram in the same way, writing the probability of each single outcome on the branches of the tree.

Consider the case where a bag contains three green marbles and two white marbles. A marble is drawn, its colour noted and it is then replaced in the bag. A second marble is then drawn. We could draw a tree diagram as shown on the right.





Using a probability tree simplifies the diagram. In a single drawing of the marble P(green) $= \frac{3}{5}$ and P(white) $= \frac{2}{5}$. These probabilities are drawn on the branches of the tree as shown below.



There are four elements to the sample space: (green, green), (green, white), (white, green) and (white, white). Each element of the sample space is not equally likely. To calculate the probability of each, we use the multiplication rule of probability.

The multiplication rule of probability states that to calculate the probability, you multiply along the branches of the tree that lead to each event. Therefore:

$$\text{P(green, green)} = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25} \qquad \text{P(green, white)} = \frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$$

$$\text{P(white, green)} = \frac{2}{5} \times \frac{3}{5} = \frac{6}{25} \qquad \text{P(white, white)} = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$$

This is the method that must be used to calculate the probability in any situation where each outcome is not equally likely.

WORKED EXAMPLE 11

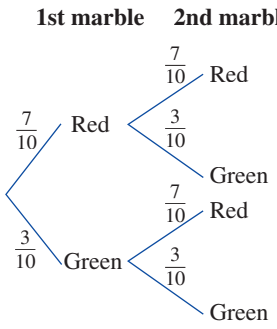
In a bag there are seven red marbles and three green marbles. A marble is drawn, its colour noted and it is then replaced in the bag. A second marble is then drawn. Find the probability that both marbles are red.

eBookplus
eLesson eles-0735 Tree diagram using multiplication principle

THINK

1 Draw the probability tree.

WRITE



2 Calculate the probability by multiplying along the branches.

$$P(\text{red, red}) = \frac{7}{10} \times \frac{7}{10}$$
$$= \frac{49}{100}$$

When asked to find the probability of an event that can occur in several ways, we need to use the addition rule of probability. The addition rule for probability states that for an event that can occur in several ways, the probability is the sum of the probabilities for each way that the event can occur.

WORKED EXAMPLE 12

In a barrel there are four blue counters and six red counters. A counter is drawn, its colour noted and a second counter is drawn. The first counter is not replaced in the barrel before the second counter is drawn. Find the probability that:

a a blue counter is drawn, followed by a red counter

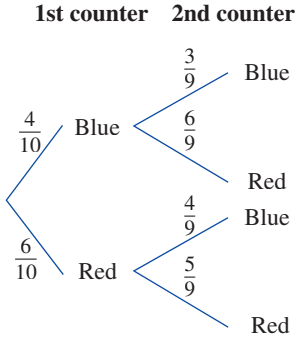
b two counters of a different colour are drawn.

eBookplus
eLesson eles-0736 Using the addition principle

THINK

1 Draw the probability tree.
- If the first counter is blue, three blue and six red counters remain in the bag.
- If the first counter is red, four blue and five red counters remain in the bag.

WRITE



2 **a** Multiply along the white, red branches to calculate the probability.

a $P(\text{blue, red}) = \frac{4}{10} \times \frac{6}{9}$
$= \frac{4}{15}$

3 **b** This outcome can occur in two ways. Add the probabilities (blue, red) and (red, blue).

b $P(\text{different colour})$
$= P(\text{blue, red}) + P(\text{red, blue})$
$= (\frac{4}{10} \times \frac{6}{9}) + (\frac{6}{10} \times \frac{4}{9})$
$= \frac{4}{15} + \frac{4}{15}$
$= \frac{8}{15}$

We must read each example carefully to see if the probabilities change throughout the experiment. In many cases we do not need to examine each possible outcome. In some examples we consider only one outcome. The branches of the tree then show if this outcome occurs or not.

WORKED EXAMPLE 13

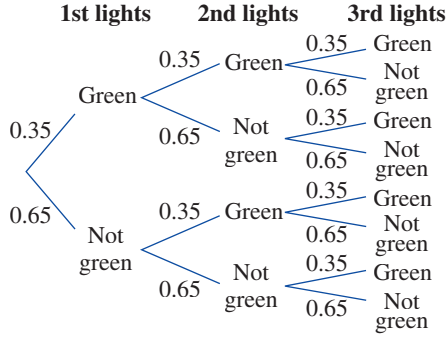
eBookplus
eLesson
eles-0737
Complementary events 1

Along a road there are three sets of traffic lights. The probability of catching a green light is 0.35. Calculate the probability of catching all three green lights.

THINK

1 Draw a probability tree. We do not need to consider if the light is red or amber, only whether it is green or not green.

WRITE



2 Calculate the probability by multiplying along the green branches.

$P(\text{three green lights}) = 0.35 \times 0.35 \times 0.35$
$= 0.042\,875$

Complementary events

Complementary events are two events that account for all possible outcomes of an experiment. For example, when rolling a die the complement of rolling a number less than three is to roll a number greater than two. The sum of the probability of an event and its complement is one.

It is often easier to calculate the probability of the complement rather than that of the event itself. We can then subtract the probability of the complementary event from one.

WORKED EXAMPLE 14

Three dice are rolled. What is the probability of rolling at least one six?

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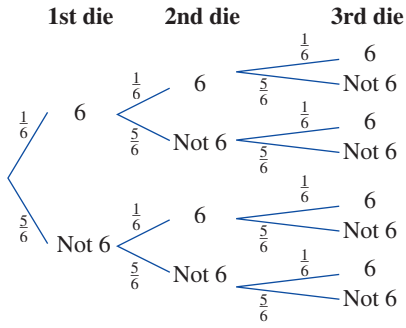
eLesson
eles-0738
Complementary events 2

THINK

1. Draw the probability tree. (We need to draw the tree with only two outcomes as we are concerned only with whether we get a 6 or not.)

WRITE



2. The complement to getting at least one six in three rolls is getting no sixes in three rolls.
3. Subtract the complement from one to find the probability.

$$\begin{aligned} P(\text{at least one six}) &= 1 - P(\text{no sixes}) \\ &= 1 - \left(\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}\right) \\ &= 1 - \frac{125}{216} \\ &= \frac{91}{216} \end{aligned}$$

REMEMBER

1. If each outcome is not equally likely, draw a probability tree with the probability of each single event on the branches.
2. To calculate a probability, multiply along the branches that give the required outcome.
3. If an outcome can be obtained in two or more ways, add the probability of each.
4. Read each question carefully to see if the probabilities change during the experiment.
5. Consider carefully what outcomes you need to include in your tree. You may need only to consider if one event occurs or not.
6. For questions that involve finding 'at least one', use the complementary event method.
7. The sum of the probability of an event and its complement is one.

EXERCISE

6D Probability trees

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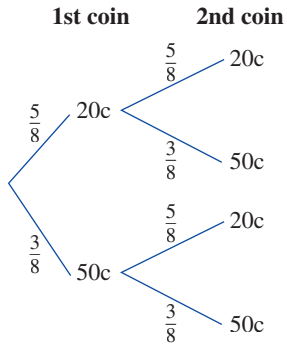
Digital doc
SkillSHEET 6.6
doc-1368
Determining complementary events

1 WE11 In a purse there are five 20-cent coins and three 50-cent coins. A coin is selected from the purse and replaced, and then a second coin is selected. The probability tree on the right is drawn for this experiment. Find the probability that the two coins drawn are both twenty cent pieces.



2 In a barrel there are four white marbles and five black marbles. Two marbles are drawn, the first being replaced in the barrel before the second one is drawn.

a Draw the probability tree for this situation.

b Find the probability for each member of the sample space.

3 WE12 A hand of five cards contains three kings and two queens. A card is chosen and then returned before a second card is chosen. Find the probability that:

a a queen is chosen followed by a king

b a king and a queen are chosen.

4 WE13 Jia is a shooter with an 80% chance of hitting a target. If he has three shots at a target, find the probability that:

a he hits with all three shots

b he hits with exactly two shots.

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Digital doc
SkillSHEET 6.7
doc-1369
Calculating the probability of a complementary event

5 A raffle has 100 tickets with two prizes. Kevin buys five tickets. Find the probability that:

a Kevin wins 1st prize

b Kevin wins both prizes

c Kevin does not win a prize

d Kevin wins exactly one prize.

6 MC A bag contains four black and six white marbles. Two marbles are drawn from the bag one after the other. If the first marble drawn is black, the probability that the second marble drawn is white is:

A $\frac{4}{9}$ **B** $\frac{2}{5}$ **C** $\frac{2}{3}$ **D** $\frac{3}{5}$

7 MC A coin is biased such that the probability of it landing heads is 0.6. The coin is tossed three times. Which of the following outcomes has the greatest probability of occurring?

A Tossing three heads

B Tossing two heads and one tail

C Tossing one head and two tails

D Tossing three tails

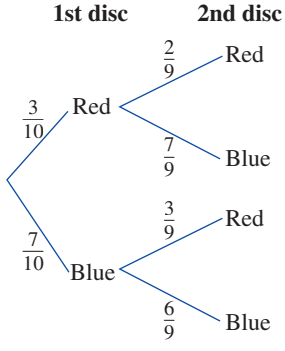
8 A box contains three red and seven blue discs. Two discs are chosen from the box. The probability tree for this experiment is shown on the right. Find the probability of selecting:

a two red discs

b two blue discs

c two discs the same colour

d two discs of a different colour.



9 The names of eight boys and five girls are placed into a hat. Two people selected from the hat are to represent the school at a function.
- **a** Use a probability tree to find the sample space for this experiment.
- **b** Find the probability of:
  - **i** two boys being chosen
  - **ii** two girls being chosen
  - **iii** one boy and one girl being chosen.

10 There are 25 students in class 12R and 24 students in class 12S. Two students are to be chosen at random to attend a study skills course. Find the probability that the two students chosen are:
- **a** from the same class
- **b** from different classes.

11 In a basket there are 15 balls, of which five are blue. Two are selected at random from the basket. Find the probability that:
- **a** two blue balls are selected
- **b** no blue balls are selected
- **c** exactly one blue ball is selected.

12 The probability that I will need to stop at a set of traffic lights is 0.55. If I twice travel through this set of lights, what is the probability of:
- **a** having to stop both times?
- **b** not having to stop either time?

13 WE14 Greg has an 80% chance of passing each Maths test. During the term he will need to sit four tests.
- **a** Find the probability that Greg will pass all four tests.
- **b** Find the probability that Greg will fail at least one test.

14 A navy ship carries surface-to-air missiles with a probability of hitting a target of 0.9. Two missiles are fired at an enemy warplane. Find the probability that the warplane escapes without being hit. (*Hint:* For the plane to escape, both missiles that are fired must miss the target.)

15 In a certain town it is known that four-fifths of all school students have been immunised against measles. For a medical test, four students need to be chosen of which at least one must have been immunised and at least one must not have been immunised. Find the probability that if four students are chosen at random:
- **a** at least one will have been immunised
- **b** at least one will not have been immunised.

16 MC Veronica rolls three dice. To win the game she needs to throw at least one six. Which of the following will give the probability of throwing at least one six?

A 1 – P(three sixes) **B** 1 – P(two sixes)
C 1 – P(one six) **D** 1 – P(no sixes)

17 There are 2 classes in Year 12:
Class 12A has 15 boys and 10 girls.
Class 12B has 12 boys and 18 girls.
The principal chooses a student to make a speech by first choosing a class at random followed by a student at random from the chosen class. Find the probability that the student chosen is:
- **a** from class 12A
- **b** a boy from class 12B
- **c** a girl.

18 In a radio contest, to win $10 000 in prize money the contestant is faced with five money bags. Each money bag has 10 coins in it. To win, the contestant chooses a bag and then chooses a coin from that bag. If the coin has the station logo on it, the contestant wins.
Bag 1 has one winning coin.
Bag 2 has three winning coins.
Bag 3 has seven winning coins.
Bags 4 and 5 have two winning coins.
Find the probability of the contestant winning the $10 000.

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Digital doc
WorkSHEET 6.2
doc-1370

19 A missile that is fired from the ground has a 0.8 chance of hitting its target. A missile fired from a plane has a 0.4 chance of hitting a target. A missile is fired from both ground and air at separate targets. Find the probability that:
a both hit their target **b** one hits its target **c** at least one hits its target.

Further development

20 A group of students is made up of 6 girls and 4 boys. Two students are to be selected to represent the group on the student representative council. They decide to write all names on identical pieces of paper, put them in a hat and choose two names randomly.
a Show the selections on a tree diagram. (*Hint:* The probabilities for the second selection change.)
b Find the probability of 2 boys being selected.
c Find the probability of 2 girls being selected.
d Find the probability of selection of 1 boy and 1 girl.
e Are the events '0 boys', '1 boy' and '2 boys' equally likely?

21 Rhonda is planning to watch three rugby league games on one weekend. She has a choice of two games on Friday night: (A) Broncos vs Cowboys and (B) Bulldogs vs Knights. On Saturday, she can watch one of the three games: (C) Eels vs Dragons, (D) Panthers vs Raiders and (E) Rabbitohs vs Roosters. On Sunday, she also has a choice of three games: (F) Sharks vs Storm, (G) Titans vs Warriors and (H) Wests Tigers vs Sea Eagles.
a To determine the different combinations of games Robyn can watch, she draws a tree diagram using codes A to H. Suggest a sample space for Robyn's selections.
b Rhonda's favourite team is Dragons. What is the probability that one of the games Robyn watches involves Dragons?
c Rhonda has a good friend that plays for Storm. What is the probability that Rhonda watches both the matches involving Dragons and Storm?

22 Dean is an archer. The experimental probability that Dean will hit the target is $\frac{4}{5}$.

a What is the probability that Dean will hit the target on two successive attempts?

b What is the probability that Dean will hit the target on three successive attempts?

c What is the probability that Dean will not hit the target on two successive attempts?

d What is the probability that Dean will hit the target on the first attempt but miss on the second attempt?

23 The probability that John will be late for meetings is $\frac{1}{7}$ and the probability that Phil will be late for a meeting is $\frac{3}{11}$. What is the probability that:

a John and Phil are both late?

b neither of them is late?

c John is late but Phil is not late?

d Phil is late but John is not late?

24 Based on her progress through the year, Karen was given a probability of 0.8 of passing the Physics exam. If the probability of passing both Maths and Physics is 0.72, what is her probability of passing the Maths exam?

25 Suresh found that on average, he is delayed 2 times out of 7 at Sydney's airport. Rakesh made similar observations at Melbourne's airport, but found he was delayed 1 out of every 4 times. Find the probability that both Suresh and Rakesh will be delayed if they are flying out of their respective airports.

SUMMARY

Tree diagrams

- A tree diagram is used in any probability experiment where there is more than one stage to the experiment.
- The sample space can be determined from a tree diagram by following the paths to the end of each branch.
- The probability of an event can then be calculated by the probability formula:

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Counting techniques

- The number of ways that n objects can be arranged in order is:
  $n!$ or $n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$.
- In an ordered selection, a number of objects are chosen and are arranged in order. The number of ordered selections can be calculated by multiplying the number of first choices that can be made by the number of second choices possible and so on until all choices have been included.
- In an unordered selection, the order in which the objects have been chosen is not important. The number of unordered selections that are possible is calculated by dividing the number of ordered selections by the number of ways the ordered selection can be arranged.
- Once the number of selections has been determined, the probability of particular selections can be determined.

Probability trees

- When each outcome is not equally likely, you draw a probability tree.
- The probability of each outcome is written on each branch of the tree.
- To calculate any probability you multiply along the branches.

CHAPTER REVIEW

MULTIPLE CHOICE

1 Which of the following is an example of an ordered selection?
- **A** A team of four people is chosen from a group of 12.
- **B** Two representatives from a class of 30 students are elected to the SRC.
- **C** A class of 30 students elect a class captain and vice-captain.
- **D** From a barrel of 44 balls, six are chosen.

2 Six people are arranged in a line. The number of ways in which this can be done is:

A 6 **B** 12 **C** 120 **D** 720

3 In a race there are six runners. In how many ways can the first three places be filled?

A 6 **B** 12 **C** 120 **D** 620

4 A group of six people consists of Darren, Shintaro, Jim, Damien, John and Allan. From these six people a group of three is chosen. The probability of choosing Darren, Jim and John is:

A $\frac{3}{6}$ **B** $\frac{1}{20}$ **C** $\frac{1}{12}$ **D** $\frac{1}{120}$

SHORT ANSWER

1 Two coins are tossed in the air.
- **a** Draw a tree diagram.
- **b** Use the tree to list the sample space for this experiment.

2 The digits 5, 7, 8 and 9 are used to form a two-digit number. Use a tree diagram to list the sample space if:
- **a** no digit can be used more than once
- **b** digits can be repeated.

3 There are three births in the maternity ward of a hospital. Calculate the probability that the babies are:
- **a** all boys
- **b** two boys and a girl
- **c** more girls than boys.

4 A two-digit number is formed using the digits 4, 6, 7, 8 and 9. No digit is allowed to be repeated.
- **a** Use a tree diagram to list the sample space.
- **b** Find the probability that the number formed is:
  - **i** 86
  - **ii** odd
  - **iii** greater than 65.

5 In a barrel there are three black marbles and three white marbles. A marble is drawn and its colour noted, and it is then replaced in the barrel. A second marble is then drawn. Find the probability of selecting:
- **a** two marbles of the same colour
- **b** at least one black marble.

6 A rowing crew has eight rowers. In how many different ways can the crew be seated in the boat?

7 From the rowing crew of eight, a captain and vice-captain are to be selected. Calculate the number of different ways the captain and vice-captain can be selected.

8 From the rowing crew of eight, four are to be chosen to crew a four-person boat. How many crews of four can be chosen from the group of eight?

9 From the digits 1, 2, 3, 4 and 5:
- **a** how many five-digit numbers can be formed if repetition is not allowed?
- **b** how many three-digit numbers can be formed if repetition is not allowed?

10 The letters D, S, T, U and Y are shuffled and placed in a line on a table. Calculate the probability that the word STUDY is formed.

11 Two students from Richard, Sandra, Talia and Ingo have to make a speech. They draw straws to see who will go first and second.
- **a** How many different ways can the first and second speaker be arranged?
- **b** What is the probability that Ingo speaks first and Talia speaks second?

12 Six teams A, B, C, D, E and F contest a basketball competition. The top four sides play in the semi-finals, and later two will contest the grand final.
- **a** In how many different ways can the top four sides be arranged?
- **b** What is the probability that the top four teams finish D, C, F and A?
- **c** How many pairs of teams is it possible to meet in the grand final?
- **d** What is the probability of A playing B in the grand final?
- **e** What is the probability that C plays in the grand final?

13 Zita is doing an exam when she realises that she has almost run out of time. She has not answered the last 10 questions.
- **a** If each question requires True or False as an answer and Zita guesses each answer, what is the probability that she guesses all 10 correctly?
- **b** If each question is multiple choice and requires the choice of (A), (B), (C) or (D), what is the probability that Zita will guess all 10 correctly?

14 In a bag there are three red marbles and two green marbles. Two marbles are drawn in succession without replacement. Find the probability that the two marbles drawn are:
- **a** both red
- **b** both green.

15 In a box there are six batteries. Two of the batteries are flat. If two are chosen from the box, find the probability that both batteries are charged.

16 The probability that a set of lights show green is $\frac{2}{5}$. If I pass through this set of lights three times, find the probability that:
- **a** I catch three green lights
- **b** I catch at least one green light.

17 In a tennis match it is noticed that Roger Federer gets 70% of serves in play. If he has two serves, find the probability that he gets at least one into play.

18 One in every eight light bulbs are faulty. If I buy three light bulbs, find the probability that none are faulty.

EXTENDED RESPONSE

1 Three coins are tossed in the air.
- **a** Draw a tree diagram to list the sample space.
- **b** Use your tree diagram to calculate the probability of tossing two heads and one tail.
- **c** Calculate the probability of tossing at least one head.

2 A basketballer has a probability of 0.4 of landing a three point shot. The basketballer has two shots at the basket.
- **a** Draw a probability tree showing all possible results of the two shots.
- **b** Calculate the probability that the basketballer:
  - **i** lands both shots
  - **ii** lands exactly one shot
  - **iii** lands at least one shot.

eBookplus ACTIVITIES

Are you ready?

Digital docs. *(page 200)*
- SkillSHEET 6.1 (doc-1361): Listing the sample space.
- SkillSHEET 6.2 (doc-1362): Informal description of chance.
- SkillSHEET 6.3 (doc-1363): Equally likely events.
- SkillSHEET 6.4 (doc-1365): Fundamental counting principle.
- SkillSHEET 6.5 (doc-1367): Single event probability.
- SkillSHEET 6.6 (doc-1368): Determining complementary events.
- SkillSHEET 6.7 (doc-1369): Calculating the probability of a complementary event.

6A Tree diagrams

eLessons
- WE1 eles-0732: Drawing a tree diagram. *(page 201)*
- WE2 eles-0733: Tree diagram with restrictions. *(page 202)*
- WE3 eles-0734: Tossing a coin and rolling a die. *(page 202)*

Digital docs
- SkillSHEET 6.1 (doc-1361): Listing the sample space. *(page 203)*
- SkillSHEET 6.2 (doc-1362): Informal description of chance. *(page 203)*
- SkillSHEET 6.3 (doc-1363): Equally likely events. *(page 203)*
- Spreadsheet (doc-1364): Tree diagrams. *(page 203)*

6B Counting techniques

Tutorial
- WE7 int-2424: Learn to select randomly from a group. *(page 207)*

Digital docs
- SkillSHEET 6.4 (doc-1365): Fundamental counting principle. *(page 207)*
- WorkSHEET 6.1 (doc-1366): Apply your knowledge counting to problems. *(page 208)*

6C Probability and counting techniques

Tutorial
- WE10 int-2425: Learn to calculate a probability. *(page 210)*

Digital docs
- SkillSHEET 6.5 (doc-1367): Single event probability. *(page 211)*

6D Probability trees

eLessons
- WE11 eles-0735: Tree diagram using multiplication principle. *(page 214)*
- WE12 eles-0736: Using the addition principle. *(page 214)*
- WE13 eles-0737: Complementary events 1. *(page 215)*
- WE14 eles-0738: Complementary events 2. *(page 216)*

Digital docs
- SkillSHEET 6.6 (doc-1368): Determining complementary events. *(page 217)*
- SkillSHEET 6.7 (doc-1369): Calculating the probability of a complementary event. *(page 217)*
- WorkSHEET 6.2 (doc-1370): Apply your knowledge of probability to problems. *(page 219)*

Chapter review

- Test Yourself (doc-1371): Take the end-of-chapter test to test your progress. *(page 223)*

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